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Mathematical Modeling and Simulation with Animation of Ball Balancing Robot

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Abstract: The characteristics of Ballbot are a self-balanced and Omni-directional mobility robot. The Ballbot is self-balanced on a single ball with only one contact point with the ground. In this work, a Ballbot system is designed for the dynamical model in order to develop a linearized model which describes the dynamics of the system by a simpler set of equations. Due to the complex dynamic behaviour of the system, the equations of motion are derived using the Lagrangian method. It consists of non-linear ordinary differential equations (ODE). Furthermore, this research proposes the dynamics of the Ballbot with the Euler-Lagrangian formulations in order to develop the model in terms of the system's physical parameters without resorting to the numeric solution. A Characteristic of developed mathematical framework is such that coupled equations are systematically eliminated. After modeling, the Ball Balancing Robot has been controlled and balanced by using a Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) Controller. Kalman filter has been used along with both the controllers to impose more robustness, stability and to counter unwanted noise. PID and LQR controller is used to correct and to further provide stability to the ball balancing robot. All this stuff has been implemented via MATLAB programming which features simple discretized mathematical equations. Later controlling, simulation has been done through MATLAB.

Keywords: - *Ballbot, Ball balancing robot, Kalman Filter, Lagrangian Method, Linear Quadratic Regulator-LQR, Proportional Integral Derivative-PID.*

I. Introduction

The Ball balancing robots is represented and executes its functional like an inverted pendulum. Such types of robots are dynamically stable. Corresponding designs are intended to be balanced on the spherical wheel which is seldom. Contact point being single along the ground, have versatile applications and rendering as Omni-directional, organic and maneuverable in motion. Ball balancing robot has various advantages over the wheel mounted robots. It is suitable for navigability in narrow, crowded and dynamic environments. Ball balancing robots are able to complete given task by carrying different types of loads.

Balancing robots possess pertaining dynamic properties that can be quantified for carrying out correct motion with precision. Balancing robots should have enough dimensions to interact with people at the level of human vision, small enough to easily adjust cumbersome surroundings, and they should move with enough pace and long lasting ability to be comparable to those of humans.

The structure of Ball Balancing Robots consists of the three Omni-wheels with motors, a ball & the body which is balanced on the ball.

Fig. 1.1 Rezero Ball Balancing Robot

II. Related Works

The first BBR is developed in 2006 by Prof. Ralph Hollis at Carnegie Mellon University (CMU), Pittsburgh, in the United States and it was patented in 2010. The CMU Ball balancing robots are structured as of dimensions comparable to the human, in all respect. Prof. Hollis with his Colleague illustrated that ball balancing robots respond to any of the disturbances quickly and rapidly with precision. Disturbances may be kicks, collisions with walls and furniture, shoves to name a few; at CMU. [9]

They proved that a different sensational human-robot physical interactions can be produced with ball balancing robots and further ended with planning and control algorithms to achieve quick, dynamic and attractive motions with the ball balancing robots. Their demonstrations further report applications involving point to point control in surveillance tasks while maintaining autonomously navigating the stuff involved. The robot was shaped to be human dimensions approximately with the intention to render it to interact with man. This was followed by arms, which were put in place to the BBR. Structure of the designed BBR then has legs which were three in number for stability under static conditions and further was having the driving mechanism which consisted of four rollers: two active ones which were DC motors driven. Their purpose was for ballbot and the other two being spring-loaded passive ones which were positioned against the drive rollers. Later was supposed to induce a force to the ballbot to ensure contact between the ball and drive rollers. This fact ensured that the robot was not free to rotate around a spinning axis which was vertical. [2] In CMU Ball balancing robots, a pair of two degrees of freedom (DOF) arms was augmented in 2011.

Thus it was the first attempt till then making it the first ball balancing robots across the globe in an ongoing decade. Later, the two rollers which were passive got removed and two active rollers have taken their place. Their purpose was obvious as different friction entities led in to and fro motion and further a drive system was provided for rotation around the spinning axis previously described. This implies that totally 5 DC motors were required, which seemed to be expensive. A lot of work was carried out and is in progress yet over this type of BBR.

The Japan-based university developed another BBR in 2008. Prof. Kumagai and allied research group elaborated the ability of the ball balancing robots to transfer loads and could be utilized for handy transportation. They developed the tiny ball balancing robots in number and proved their use in underlying similar applications. [6]

The one, which now we are discussing is tiny compared to those of CMU, but particular stuff about former is that it can perform the same motion with only three motors connected to three Omni-wheels. Underlying motion about which we are discussing is rotation around the spinning axis. Further, this robot can take up the loads starting at 10 kg. A group of researchers independently presented the design for a human-reliable ball balancing robots based wheelchair that sets equilibrium on a basketball popular as B. B. Rider. However, they demonstrated facts theoretically without experimental database [4].

Hungary based researcher, László Havasiseldom introduced different BBR and named it as ERRO Sphere. The robot did not confirm to be performing its duties as expected and hence it was halted that end since then. The University of Adelaide (UA) in Australia developed a BBR using LEGO Mind storms NXT in 2009. It is a tiny robot having dimensions of 20 cm in height, completely structured of LEGO. It has two wheels which drive the ball. Tomás Arribas of Spain invented first ball balancing robots. He used LEGO Mind storms NXT kind of utility during 2008. He used excel based programming skills in his simulations. As part of the research carried out inside the Space Research Group of the University of Alcalá (SRG-UAH), Spain, the work team, specialized in optimal control and planning applied to dynamic systems involving non linearity, published their outcomes in 2012 in the article titled "A Monoball Robot Based on LEGO Mindstorms" Mathematical aspects were detailed in the article. This was further extended to trajectory control as a reference to non-linear control systems which are unstable. [9]

Taking inspirations from Tomas Aribas, Yorihisa Yamamoto (Japan) extended a ball balancing robots using LEGO Mind storms NXT via a demo using MATLAB based optimization. A group of mechanical engineering students at University of Adelaide (Australia) developed both a LEGO Ball balancing robots and a full-scale Ball balancing robots in 2009. Students from ITMO University (Russia), manufactured a Ball balancing robots based on Lego NXT robotics kit with a back up of the algorithm and which performed stability having only two actuators employed.[36] A lot of video material is available freely across the globe which describes several ball balancing robots doing service around the globe. Some of them use LEGO Mind storms NXT as already discussed while other talks about custom designs having Omni-wheels in place for ball actuation. [1]

Ball robots are also making their impression in the Science of Fiction World. Pixar's Wall-E (2008) movie demonstrates-*O* (Microbe Obliterator) which is a typical ball balancing robots. Syfy's Caprica TV series (2010) featured *Serge*, a ball balancing robots butler robot.

Taiwan based university (NCHU) in manufactured a BBR in 2012, identical to the one of the TGU. It was having three multi wheels known as Omni-wheels and is similar in dimensions to that of TGU.

Rezero was developed in 2010 at ETH Zurich in Switzerland. Omni wheels were similar here as well. It has a robust dynamics, and determined to have a linear velocity within the range of 2 m/s and can be tilted to an angle of 20. [3]

III. Structure of Ball Balancing Robot

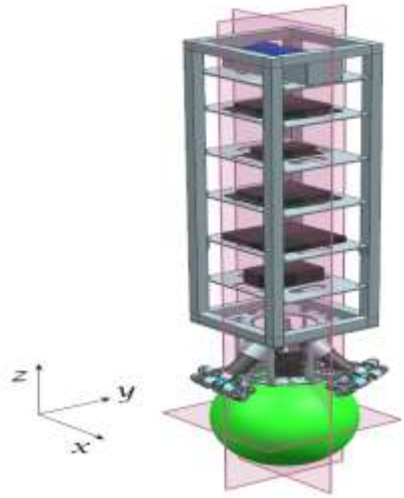

Fig. 3.1 Axial Plans of BBR [6]

The most fundamental design parameters of a ball balancing robots are its height, mass, its center of gravity and the maximum torque. The selections of these parameters evaluate the robot's moment of inertia, the maximum rolling, pitching angles and thus its dynamic and kinematic performance.

The maximum velocity is dependent on the power of actuator and its inbuilt capability. Apart from the friction coefficients, all other parts play the major role in force transmission and in control system design. These kinds of stuff play key roles. Also, important analysis has to be done over the ratio of the moment of inertia of the robot body, its ball, kinematics, underlying dynamics, mechanics-based intuition and boundary conditions, prescribed displacements and like in order to prevent undesired ball spin, for instance in yawing [3].

The problem such as that of actuation of a sphere, there are lot of mechanisms that can be put in place and each of mechanism is found to be suitable at the particular thing or the other. For instance, CMU Ball balancing robots uses the mechanism with an inverse mouse-ball drive. Contrary to the traditional mouse ball that usually drives the rollers of a mouse, the inverse mouse-ball drive uses rollers to yield the ball producing motion. Accordingly, inverse mouse-ball uses four rollers to propel the ball and each roller is in connection with a seldom electric motor. To have yaw motion, bearings are used in the CMU Ball balancing robots [1]. Apart from that, slip-ring assembly and a separate motor which accomplishes the spinning of the body on top of the ball are also in place. The similar mechanism is also used in LEGO Ball balancing robots. Additionally, normal wheels to drive the ball instead of rollers were used [2].

IV. Methodology & Mathematical Modelling

A dynamical model is required to get a deeper insight into the system. Also to control the BBR, equations of motion are required, which can be derived using a dynamical model of the BBR.

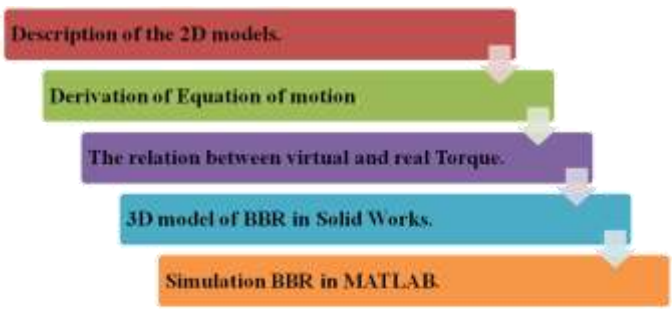

Fig. 4.1 Methodology

Assumptions-
Here are some assumptions in modeling.

Seldom vertical and horizontal planes:-
Both the planes are found to have an independent way and hence the kinematic description is separately considered.

Rigid behavior bodies/floor:-
The total system comprises two rigid entities or bodies, e.g. the ball and the body of the robot along with drive assembly and the multi wheels (Omni wheels) affixed to it. This reduces coupled equations of motion which yields equations of motion in a simple form to be handled by MATLAB alone. The word rigid bodies avoid deformations involved and thus multi body approach is eliminated. Furthermore, as deformation of the floor is ignored, secondary forces are eliminated

Friction:-
All the drives are assumed to be frictionless.

Slip less condition:-
It is assumed that kinematic restrictions are such that slipping of the ball along the floor is not possible and same is true between the Omni wheels and ball. This supports the following arguments:-

a) The torques induced of the motors are confined to values such that slipping of the ball and the Omni-wheels are not allowed.
b) Static friction is rendered high enough so that all the instantaneous centers remain intact and inertias are not required to be around other unknown axes. Specifically, kinematic friction which is high prevents the ball undergoing rotation around the spinning axis is confirmed by putting delimiters over the torque along the spinning axis.
c) No lifting out of the ground occurs for the ball and hence is always make contact with the floor. This avoids computations with potential energy due to the lifting of a robot out of the ground. Also, the floor is rough by an appropriate amount to prevent slipping. Further care has been taken that there are no obstacles.

Flattening of the floor:-
It is presumed that the floor, which lets the robot moves, is perfectly horizontal, which implies that the ball is without potential energy.

Very low time-lapse:-
It is ascertained that the time lapse in the successive measurements of the sensors and the control of actuators is damp minimum.

Omni-wheels:-
It is further presumed that the 2-row Omni-wheels, which obviously have contact which is the considerable need to be modeled as 1-row Omni-wheels having a single contact point.

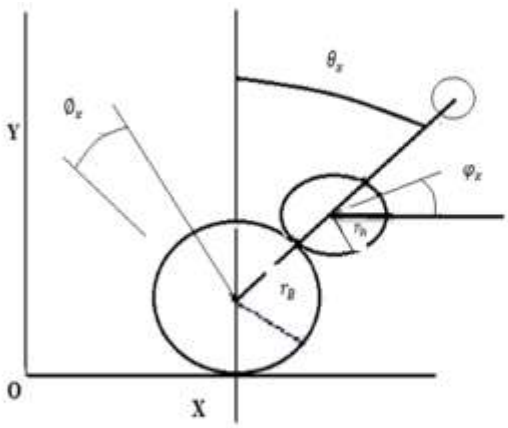

Fig. 4.2 Co-ordinate Sytem of BBR

Direction-cosine matrix

$$\varepsilon = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; ; \; \eta = \begin{bmatrix} \emptyset \\ \theta \\ \varphi \end{bmatrix} \; ; \; \delta = \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

Linear & Angular Velocity of BBR Body

$$V_B = \begin{bmatrix} V_{xB} \\ V_{yB} \\ V_{zB} \end{bmatrix} ; \; \nu = \begin{bmatrix} \mathrm{p} \\ \mathrm{q} \\ \mathrm{r} \end{bmatrix}$$

The rotation matrix from the body frame to the inertial frame

$$[R] = \begin{bmatrix} cos\theta \cos\Psi & sin\theta sin\varphi \cos\Psi - \sin\Psi \cos\varphi & sin\theta \cos\varphi \cos\Psi + \sin\varphi \sin\Psi \\ sin\Psi cos\theta & sin\theta sin\varphi sin\Psi + cos\varphi \cos\Psi & \sin\theta \cos\varphi \sin\Psi - \sin\varphi \cos\Psi \\ -sin\theta & \sin\varphi\, cos\theta & cos\theta \cos\varphi \end{bmatrix}$$

Angular rates transformation

$$[\omega_\eta] = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\emptyset & \sin\emptyset \cos\theta \\ 0 & -\sin\emptyset & \cos\theta \cos\emptyset \end{bmatrix}$$

V. Dynamic Analysis

External forces and Torques with the help of Lagrangian Equation.

$$\mathrm{L} = E_{translational} + E_{rotational} - E_{potential}$$

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{S}}\right) - \frac{\partial L}{\partial S}$$

Linear Motion-

$$\mathrm{L} = E_{translational} - E_{potential}$$

$$f = T \begin{bmatrix} sin\theta \cos\varphi \cos\Psi + \sin\varphi \sin\Psi \\ \sin\theta \cos\varphi \sin\Psi - \sin\varphi \cos\Psi \\ cos\theta \cos\varphi \end{bmatrix}$$

Jacobian matrix$[J]$

The equation can be linearized with help the Jacobian Matrix.

$$[J] = [W_\eta]^T [I][W_\eta]$$

$$[J] = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} cos^2\varphi + I_{zz}\sin^2\varphi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now the External angular force is torque of a motor.

Angular Lagrangian equations are,

$$\tau_B = [J][\ddot{\eta}] + \frac{\mathrm{d}}{\mathrm{dt}}[J][\dot{\eta}] - \frac{1}{2}\frac{\partial}{\partial \eta}\left((\dot{\eta}^T \, J \, \dot{\eta}\right)$$

$$\tau_B = \begin{bmatrix} \dot{p} I_{xx} \\ I_{yy}(sin\emptyset q\dot{\emptyset} + cos\emptyset\dot{q}) + I_{zz}(tan^2\emptyset sin\emptyset q\dot{\emptyset} + tan\emptyset sin\emptyset\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ sin2\emptyset\dot{\emptyset} q sec\emptyset(I_{zz} - I_{yy}) \end{bmatrix}$$

By substituting all values in the τ_B equation we get,

$$\tau_B = \begin{bmatrix} \ddot{\varphi} I_{xx} \\ I_{yy}cos\varphi(\ddot{\theta}cos\varphi - \dot{\theta}sin\varphi - \dot{\varphi}\dot{\theta}sin\varphi) + \mathrm{I}_{zz}\left(sin2\varphi\left(\frac{tan^2\varphi}{2} + 1\right)\dot{\varphi}\dot{\theta} + tan\varphi(\ddot{\theta} - tan\varphi\dot{\theta}\right) \\ 0 \end{bmatrix}$$

Angular accelerations

$$\therefore \ddot{\eta}_1 = \ddot{\varphi}$$

$$\therefore \ddot{\eta}_2 = \ddot{\theta}$$

Torque

$$\tau_1 = \mathrm{I}_{xx}[\ddot{\emptyset}]$$

$$\tau_2 = \mathrm{I}_{yy}\left(\ddot{\theta}\cos\emptyset^2 - \sin 2\emptyset\,\dot{\emptyset}\dot{\varphi}\right) + \mathrm{I}_{zz}\left[\left(\tan\emptyset \sin\emptyset^2 \dot{\theta} + \dot{\emptyset}\dot{\varphi}(\sin 2\emptyset - \tan\emptyset \sin\emptyset^2)\right) + \sin\emptyset^2 \ddot{\theta}\right]$$

Accelerations

$$\ddot{\varepsilon}_1 = \frac{T}{m}(\sin\theta \cos\emptyset) + \frac{k}{m}(x)$$

$$\ddot{\varepsilon}_2 = \frac{T}{m}(-\sin\emptyset) + \frac{k}{m}(y)$$

Fig. 3D model of BBR

$$\ddot{\varepsilon}_3 = \frac{T}{m}(\cos\theta \cos \emptyset) + \frac{k}{m}(z)$$

VI. Selection Of Suitable Controller

The Proportional Integral Derivative PID Controller is used in modern industries for automation purpose. PID controller usually minimizes the signal error by considering the present, past & future error. The Proportional-part for error at present, the Integral-part for integral the error up to the present and the Derivative-part for derivate the error up to the present time.

In Ball Balancing Robot the main purpose of the controller is to keep the body upright by balancing angle of the body. PID controller senses the signal of the error of body's orientation angle from consistent position then makes the input of a torque to the associated motor which has direct link or mechanical communication with wheels. It controls only one current kinematic description so that the wheel position goes out of control. At the same time, wheels response is no concrete and just makes their movements to equilibrate the angle due to pitch. PID inputs the amount to the wheel and controls it through backward and frontal movements. Thus pitch angle will be in control and keeping the equilibrium of BBR intact.

PID controller as its name suggests is proportional plus integral plus derivative controller. To this end meaning is that if the body is displaced by some angle due to disturbance, PID controller senses the error from mean positions and tries to control the same. It takes the control as one at a time. Meaning is that as the wheels are aligned they need to be taken care off. But as PID controller senses change in angle of the body it tries to restore. As back reaction wheels also tend to resume original position. This is achieved in incremental steps known as derivatives. But as the derivatives may be linear or nonlinear controller adjusts itself to set a proportion. Finally, Incremental positions are achieved with updations which are known as integration. Hence complete control is achieved as proportional plus derivative plus integral.

By adding the PID controller to the LQR stabilized the system, take the wheel position along with body position thus rendering the great control. As already discussed wheel angle is not controlled by PID. There it has to be controlled by somebody else. That is LQR. Hence LQR plus PID sets complete balance. For LQR to work properly, generated signal requires filtering. Otherwise, there will be an error in the sensing angle of wheels. To maintain precision Kalman Filter is required. One more reason is that the signal may have additional noise. This misjudges the incremental angle of wheel required for equilibrium by LQR. To rectify this error Kalman filter is required. To provide more robustness to a solution, the equations derived are implemented in MATLAB with some inherent system features.

VII. Selection Of Suitable Filter

Kalman Filter is used to eliminate measurement noise and helping to get a reasonable angle forecasting. The evaluated angle then processed through the Kalman filter will be useful to LQR controller to hold the stability of wheel.

Kalman Filter is based upon linearstuffs to evaluate the current scene and is not in need of a history relevant to past data set. It captures last evaluated value and present one from IMU to estimate a present processed value. Kalman filter is powerful in various aspects. It backs evaluations of past, current, and even future state and it can perform the same proliferously despite the precise nature of the modeled system is unpredictable.

VIII. Simulation

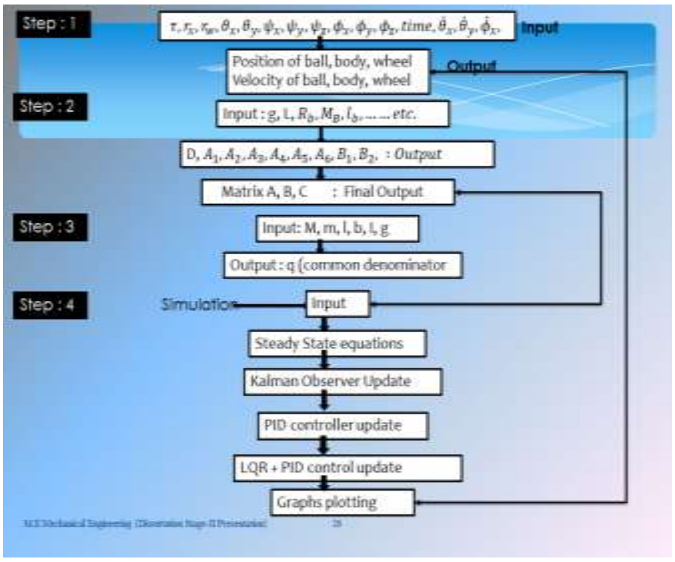

Fig. 8.1 Flow chart of execution of the program

In present simulations of ongoing work, use is made effectively of PID and LQR combination with newly developed equations in required format by MATLAB. Equations are then discretized suitably. A simple form of expressions is used effectively without coupled terms for better controller design in future technology.

IX. Results

A. Modeling Result

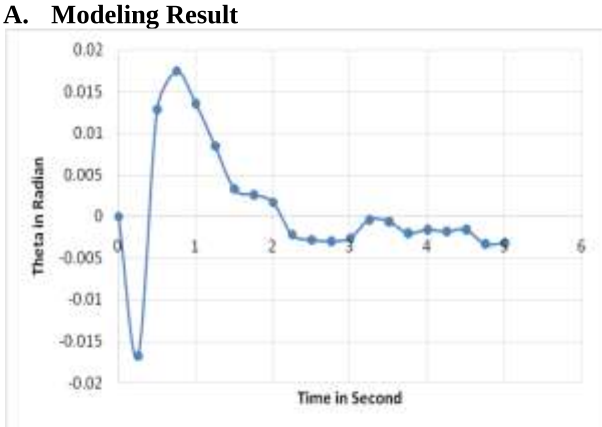

Fig. 9.1 Graph of Theta vs Time

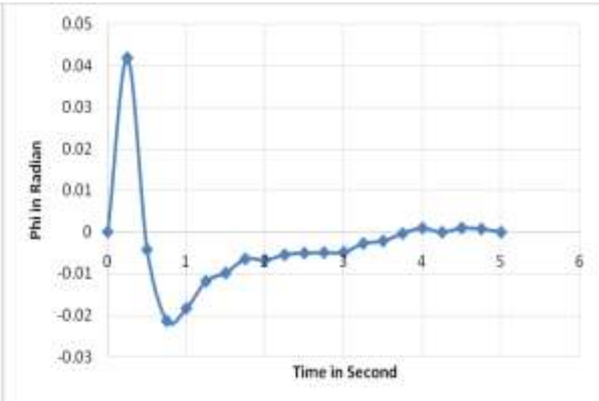

Fig. 9.2 Graph of Phi vs Time

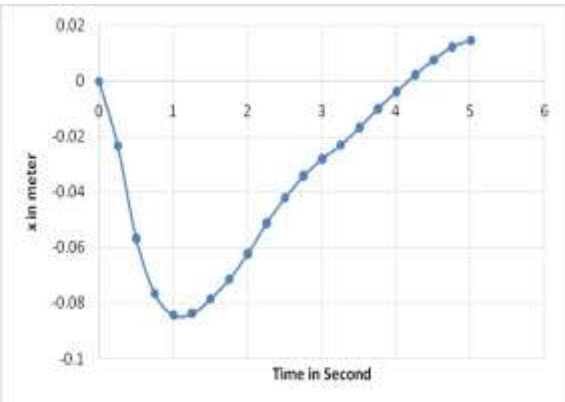

Fig. 9.3 Graph X vs Time

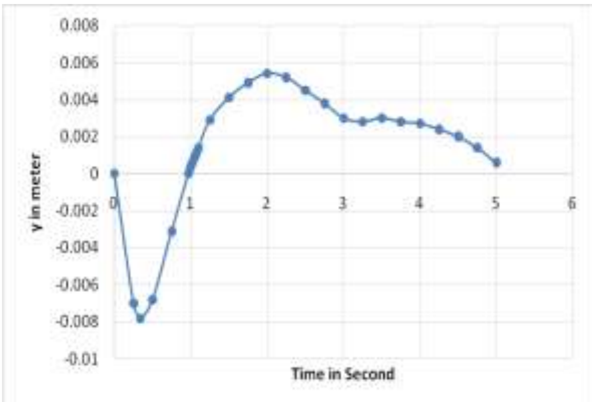

Fig. 9.4 Graph Y vs Time

In the above figure (7 & 10), angular position rises initially. Characteristic similar to sidewise leaning falling type (x and phi) slope is steep down initially and stabilizes later. All results stabilize after initial disturbance. If linear displacement increases to 0.17 units hence indirectly acceleration in the Y direction at around 1 sec., corresponding value for X-axis settles to 0.1 units. All the quantities are referred in SI units. In terms of Y-axis i.e., displacement or acceleration system restores to equilibrium due to stability smoothly & parabolic. Its inverse parabola for Y and forward or regular parabola for X. Values are finally settled to zero in 5 seconds.

B. Animation Results

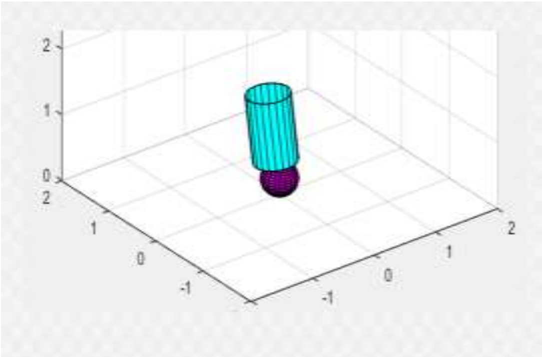

Fig9.5 Initial disturbance given to ball balancing robot.

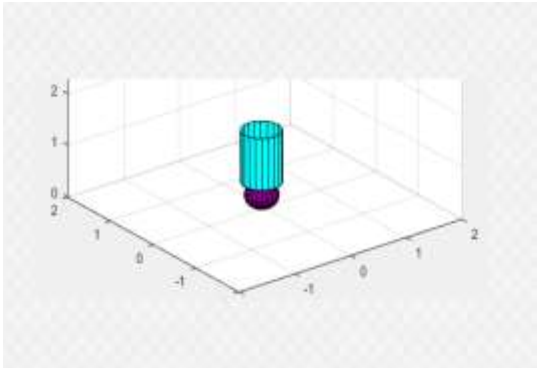

Fig. 9.6 structure of ball balancing robot gets stable

X. Conclusion

This work presents a kinematic and dynamic analysis of ball balancing robot. All necessary equations are explicitly derived from preliminary kinematics with double Plane motion analysis. All coupled terms are eliminated. As a result simplified form of equations is recast. Finally, important equations are implemented through general purpose MATLAB code. LQR and PID controllers were used to stabilize solutions with reasonable accuracy and precision. All the results are consistent and promising. This provides the better platform for controller hardware and real-time prototype of ball robot in future.

XI. Abbreviations

BBR	Ball-Balancing Robot
PID	Proportional-Integral-Derivative
LQR	Linear Quadratic Regulator
DOF	Degrees of Freedom
IMU	Inertia Measuring Unit
3D	Three-Dimensional
2D	Two-Dimensional
CMU	Carnegie Mellon University
COM	Centre of Mass

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